

no friction, and prevents the moment of inertia of the scale-pan and load interfering, by means of a rotatory motion, with the rapidity with which the pointer comes to rest when a load is put into the pan. The position of the pointer P, which revolves when a weight is placed in the scale-pan, is read off upon the spiral scale, D, which, in this specimen, we have graduated in pounds. In another of these weighing machines, shown in fig. 10, the arrangement is the same with the exception that a cylindric scale D is fixed to the end of the spring and turns with it, the pointer P fixed on the frame of the instrument points to an indication of the weight on a spiral line drawn on the cylinder D. This second arrangement allows us to employ springs whose ends have a relative motion of five or six revolutions.

We have also made certain weighing machines where the weight is placed on a pan resting on the top of a rod passing up through the spring, and attached to the bottom of the spring, but hitherto we have found that this arrangement introduces too much friction.

When a very long spring is required for any purpose, and where the weight of the spring itself would cause greater stresses on the upper part than on the lower part of the spring, it is our custom to let the breadth and thickness of the strip of material remain unaltered, but to let the diameter of the coils diminish towards the upper parts. The formulæ already given suffice to show how this may be done so as to obtain uniformity of maximum stress at all sections.

We also present before the Society a model showing a combination of bifilar and spiral spring suspension, in which great rotation and small axial lengthening or shortening are produced by an axial force.

III. "Note on the Theory of the Magnetic Balance of Hughes."

By Professor SILVANUS P. THOMPSON, B.A., D.Sc., Univ. Coll., Bristol. Communicated by Professor D. E. HUGHES, F.R.S. Received February 7, 1884.

[1.] The magnetic balance recently described by Professor D. E. Hughes* promises to be so convenient and useful an instrument in the laboratory, that the theory of its action and graduation deserves attention. In the actual instrument constructed by Hughes the graduation was empirically determined for a number of values, the remainder being found by interpolation. The instrument consists of a small suspended needle lying in the magnetic meridian provided with a zero-mark placed upon a platform in which a horizontal groove is cut magnetically east and west. In this groove the piece of iron

* "Proc. Roy. Soc.," vol. 36, p. 167.

compensator NS, which has been turned through an angle $\text{ONS}=\phi$ in order to balance the magnetic force exerted on the suspended needle at B by the specimen of iron or steel placed at A. It is required to determine the relation between the angle of turning ϕ , and the magnetic force thereby brought to bear along the axis AX at B tending to thrust back the suspended needle to its zero point. Let the length of the compensator be called $2l$, and the distance of B from M= r . It is evident that, in general, the resultant magnetic force at B due to the compensator will not be along the axis AX, but may be resolved into a part acting at right angles to that axis, and a part parallel to it. The former part will be parallel to the magnetic meridian, and, therefore, when the needle is at zero, this part will not tend to move the needle to either side. Its only action is to increase or diminish (according to circumstances) the directive force of the earth's magnetism upon the needle, and render it either more or less sensitive. The other part is that which acts along the axis and balances the magnetic force of the iron or steel bar at A. When the compensator lies at right angles to AX this component of the force vanishes by symmetry: consequently the zero of the scale lies in this line. If the compensator be turned through an angle ϕ , the component of its magnetic force in the line AX will increase to a maximum when $\phi=90^\circ$. The values of the force for different angles may be calculated as follows:—

$$\text{Call } BS=r',$$

$$BN=r'',$$

$$\text{angle SBM}=\alpha,$$

$$\text{angle NBM}=\beta.$$

Then the forces along BS and BN resolved along AB will give the following resultant F:—

$$F=m \left\{ \frac{\cos \alpha}{r'^2} - \frac{\cos \beta}{r''^2} \right\};$$

m being the strength of the pole of the compensator.

$$\text{But } \cos \alpha = \frac{r-l \sin \phi}{(r^2+l^2-2rl \sin \phi)^{\frac{1}{2}}};$$

$$\cos \beta = \frac{r+l \sin \phi}{(r^2+l^2+2rl \sin \phi)^{\frac{1}{2}}};$$

and

$$r'=(r^2+l^2-2rl \sin \phi)^{\frac{1}{2}};$$

$$r''=(r^2+l^2+2rl \sin \phi)^{\frac{1}{2}}.$$

$$\text{Whence } F=m \left\{ \frac{r-l \sin \phi}{(r^2+l^2-2rl \sin \phi)^{\frac{3}{2}}} - \frac{r+l \sin \phi}{(r^2+l^2+2rl \sin \phi)^{\frac{3}{2}}} \right\} \quad (1),$$

or, writing n for the ratio r/l ,

$$F = \frac{m}{l^3} \left\{ \frac{n - \sin \phi}{(n^2 + 1 - 2n \sin \phi)^{\frac{3}{2}}} - \frac{n + \sin \phi}{(n^2 + 1 + 2n \sin \phi)^{\frac{3}{2}}} \right\} \quad (2)$$

which for brevity we may write

$$F = \frac{m}{l^3} P$$

$$\text{where} \quad P = \frac{n - \sin \phi}{(n^2 + 1 - 2n \sin \phi)^{\frac{3}{2}}} - \frac{n + \sin \phi}{(n^2 + 1 + 2n \sin \phi)^{\frac{3}{2}}}$$

It is obvious from the mere form of this expression that it is not accurate to assume that a scale of equal degrees will represent equal increments of the magnetic force; and that no scale ordinarily used in galvanometers or magnetometers, such as a scale of sines or of tangents, will adequately serve the purpose. It is, therefore, necessary to investigate the formula to ascertain how it can be made available for the graduation of a scale for the instrument.

The expression is an awkward one for numerical calculation, owing to the fractional exponents of the denominators; but the process of calculation can be somewhat shortened by taking n as a simple integer, and the successive values of ϕ such that the sines are simple decimals. In the following tables will be found the values of the function P , calculated for several cases, the angles being in all cases expressed in degrees and decimals of degrees, not in degrees, minutes, and seconds. The object in calculating these values was to ascertain the proportions of r and l , which would enable the scale readings of the compensator to be used without the labour of constructing arbitrary interpolation tables for the calibration of the readings.

Table I.—($n=1$.)

Angle.	Nat. sine.	P calculated.
0°	0	0
5·739	0·1	0·0355
11·538	0·2	0·0726
17·457	0·3	0·1195
23·578	0·4	0·1576
30·000	0·5	0·2113
36·869	0·6	0·2794
44·427	0·7	0·3741
53·130	0·8	0·5271
64·158	0·9	0·8618
71·805	0·95	1·3280
90·000	1·0	∞

Table II.—($n=2$.)

Angle.	Nat. sine.	P calculated.
0°	0	0
10·000	0·1736	0·0444
20·000	0·3420	0·0938
30·000	0·5000	0·1568
40·000	0·6428	0·2235
50·000	0·7660	0·3369
60·000	0·8660	0·4787
70·000	0·9397	0·6534
75·000	0·9659	0·7412
80·000	0·9848	0·8168
85·000	0·9962	0·8736
90·000	1·0000	0·8888

Table III.—($n=3$.)

Angle.	Nat. sine.	P calculated.
0°	0	0
5·739	0·1	0·0107
11·538	0·2	0·0219
17·457	0·3	0·03256
23·578	0·4	0·0462
30·000	0·5	0·0604
36·869	0·6	0·0764
44·427	0·7	0·0955
53·130	0·8	0·1188
64·158	0·9	0·1484
71·805	0·95	0·1664
90·000	1·0	0·1875

Table IV.—($n=4$.)

Angle	Nat. sine.	P calculated.	P observed.
0°	0	0	0
5·739	0·1	0·005218	0·00556
11·538	0·2	0·010521	0·01136
17·457	0·3	0·010001	0·01716
23·578	0·4	0·021758	0·02271
30·000	0·5	0·027910	0·02833
36·869	0·6	0·034709	0·03488
44·427	0·7	0·041985	0·04198
53·130	0·8	0·050319	0·05136
58·211	0·85	0·054926	0·05605
64·158	0·9	0·059886	0·06061
71·805	0·95	0·065260	0·06554
90·000	1·0	0·071111	0·07290

Table V.—($n=5$.)

Angle.	Nat. sine.	P calculated.	P observed.
0°	0	0	0
5°739	0·1	0·002848	0·00267
11°538	0·2	0·005809	0·00545
17°457	0·3	0·008772	0·00848
23°578	0·4	0·011715	0·01083
30°000	0·5	0·014895	0·01468
36°869	0·6	0·018257	0·01759
44°427	0·7	0·021829	0·02183
53°130	0·8	0·025741	0·02564
58°211	0·85	0·027819	0·02645
64°158	0·9	0·030000	0·02801
71°805	0·95	0·032297	0·03081
90°000	1·0	0·034722	0·03365

The values of P set down as observed in the last column of the preceding tables were obtained by noting the deflexions produced on a tangent magnetometer, with very small needles, by a magnet placed at the requisite distance (measured horizontally at right angles to the magnetic meridian), capable of rotation in a vertical plane passing through its centre and through that of the magnetometer. The readings were then multiplied by such a constant as would make the value at 44°·427 coincide with the calculated value at the same angle. The departures at other parts of the scale are partly due to errors of observation, partly due to the fact that the distance between the poles of the magnet is less than the distance between the ends.

Table VI.—($n=6$.)

Angle.	Nat. sine.	P calculated.
0°	0	0
5°739	0·1	0·001708
11°538	0·2	0·003428
17°457	0·3	0·005163
23°578	0·4	0·006966
30°000	0·5	0·008814
36°869	0·6	0·010735
44°437	0·7	0·012749
53°130	0·8	0·014796
58°211	0·85	0·015975
64°158	0·9	0·017150
71°805	0·95	0·018348
90°000	1·0	0·019592

Table VII.—($n=10$.)

Angle.	Nat. sine.	P calculated.
0°	0	0
5·739	0·1	0·000390
11·538	0·2	0·000778
17·457	0·3	0·001169
23·578	0·4	0·001505
30·000	0·5	0·001966
36·869	0·6	0·002374
44·427	0·7	0·002784
53·130	0·8	0·003207
64·158	0·9	0·003638
71·805	0·95	0·003858
90·000	1·0	0·004090

The calculated values of P are plotted out in the two sheets of curves (figs. 2 and 3) accompanying the tables. The curves for $n=4$ and $n=5$ are given twice, being drawn again with enlarged ordinates to show on a larger scale their close approximation to straight lines up to about 50° .

[3.] From the foregoing tables, and from the curves appended, several conclusions may be immediately drawn with respect to the design of the magnetic balance. A simple inspection of the curves will show that they belong to a family in which, in general, there is a concavity near the origin, and a convexity as the limiting value is approached at the point corresponding to the 90° position of the compensator. Those curves for which $n=3$, or less than 3, show the convexity very markedly. In those for values of n higher than 7 (only one has been drawn, namely, that for $n=10$) the convexity of the upper portion asserts itself. The curve for $n=10$ approximates very nearly to a curve of sines, as indeed might be suspected from the equation.

For those values of n which lie between 4 and 6 inclusive, the first half of the curve is very nearly a straight line, so nearly so that for the curves $n=4$ and $n=5$ the values of the ordinates do not differ by 1 cent. from those which they would have for actual straight lines lying along the mean slope of the respective curves as far as 45° . In other words, for angles less than 45° the magnetic force exercised along the axis of the balance by the compensator when it is turned is *proportional*, within 1 per cent. of accuracy, to the angle through which it has been turned, provided the distance of the compensator from the needle be not less than four and not greater than five times the half-length of the compensator.

This result may be verified from equation (2) by finding what

FIG. 2.

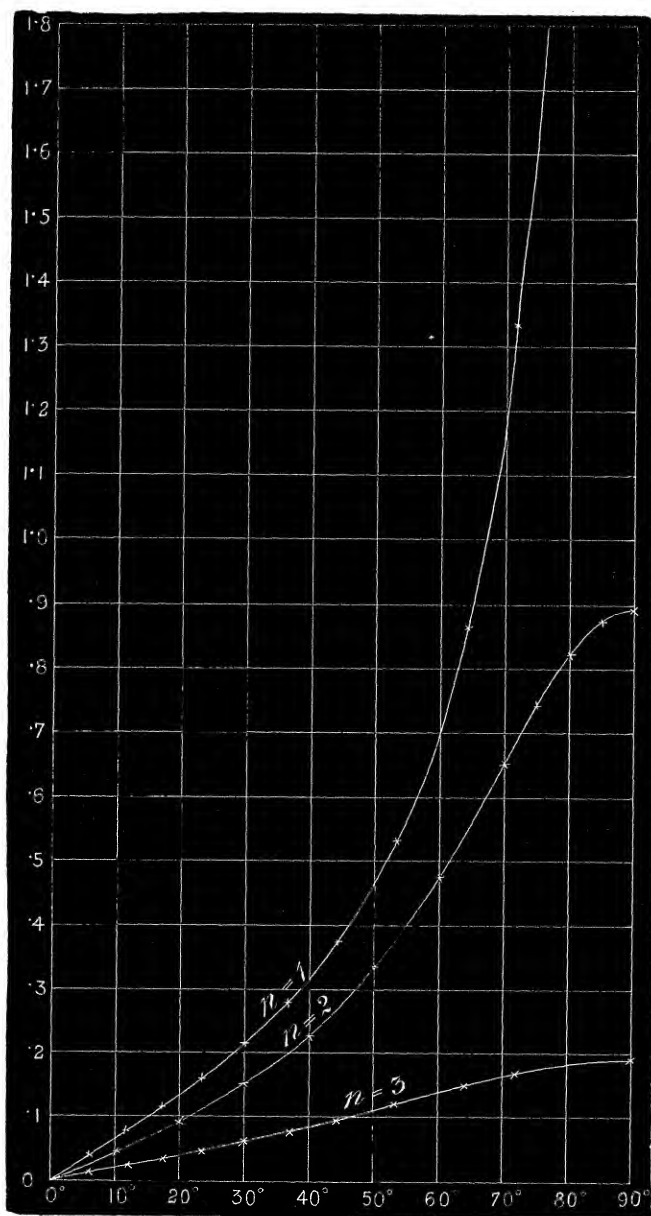
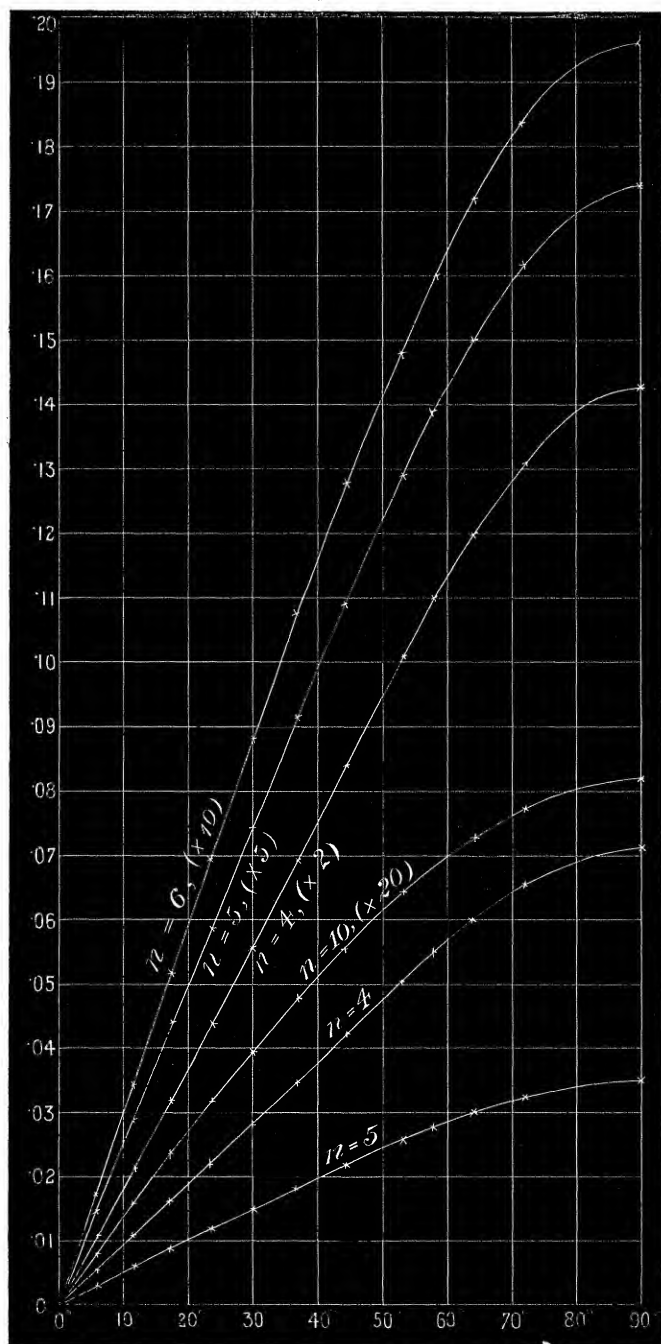


Fig. 3.



value of n will give the force when $\phi=45^\circ$ equal to $\frac{3}{2}$ of the force when $\phi=30^\circ$. Writing in the sines of these angles and equating 3 times the force at 30° to 2 times the force at 45° , we get the value $n=4.6$. Or, solving similarly for the case that the force at 60° shall be double the force at 30° , we get $n=4.28$. Here, however, the errors near the intermediate angle of 45° are not inconsiderable, exceeding 4 per cent. In any case we shall not be far wrong if we adjust the balance so that $n=4.5$, and confine our readings within 45° of zero. For purposes where great accuracy is not essential the readings might be extended as far as 60° , since the curves for $n=4$ and $n=5$, show how very little the straight line is departed from up to that point. In the original balance of Hughes, the values of r and l are about 30 centims. and 7.5 centims. respectively, so that $n=4$.

[4.] I now proceed to the consideration of the action of the compensator in affecting in different positions the sensitiveness of the needle.

In the original instrument of Hughes, the compensator turned in a horizontal plane; and therefore in every position, save only when at 90° to the zero of its scale or when end-on to the indicating needle, the force exercised by it on the needle would have a component in the magnetic meridian. As remarked above, the effect of this component would be to increase or diminish the directive force of the earth's magnetism upon the needle, according as the S pole of the compensator pointed northwards, or southwards. The sensitiveness of the indicating needle will, of course, be affected by this component; being a maximum when the component is such as nearly to astatise it. But it is evident that if the needle be nearly astatised when the compensator is at zero, it will not be so astatised when the compensator is moved right or left. The sensibility of the needle will diminish as the effective force of the compensator increases by its being turned. The calculation for the component of the force at this point is as follows:—

Calling T the part of the magnetic force at B (fig. 1) resolved at right angles to AX, we have—

$$T = m \left\{ \frac{\sin \alpha}{r'^2} - \frac{\sin \beta}{r''^2} \right\}.$$

But

$$\sin \alpha = \frac{l \cos \phi}{(r^2 + l^2 - 2rl \sin \phi)^{\frac{3}{2}}}$$

and

$$\sin \beta = \frac{l \cos \phi}{(r^2 + l^2 + 2rl \sin \phi)^{\frac{3}{2}}}$$

whence
$$T = ml \cos \phi \left\{ \frac{1}{(r^2 + l^2 - 2rl \sin \phi)^{\frac{3}{2}}} + \frac{1}{(r^2 + l^2 + 2rl \sin \phi)^{\frac{3}{2}}} \right\}.$$

Writing, as before, $r=nl$, this becomes :—

$$T = \frac{m}{l^2} \cos \phi \left\{ \frac{1}{(n^2 + 1 - 2n \sin \phi)^{\frac{3}{2}}} + \frac{1}{(n^2 + 1 + 2n \sin \phi)^{\frac{3}{2}}} \right\} \quad (3).$$

Assuming that the balance is adjusted for the case $n=5$, we get the following values for this complex function of ϕ :—

$$\phi=0, \quad T = \frac{m}{l^2} \times \cdot 01508.$$

$$\phi=30^\circ, \quad T = \frac{m}{l^2} \times \cdot 01402.$$

$$\phi=60^\circ, \quad T = \frac{m}{l^2} \times \cdot 00938.$$

$$\phi=90^\circ, \quad T = \frac{m}{l^2} \times 0.$$

The needle, therefore, if nearly astaticised when $\phi=0$, will still be nearly astaticised when ϕ does not exceed 30° ; but beyond this value the meridional component of the force falls off greatly, and the sensitiveness alters correspondingly.

[5.] *Suggested Modifications in the Magnetic Balance.*—The preceding investigation has led to the following suggestions :—

1stly. The compensator should be set so as to revolve against a vertical circle placed at right angles to the magnetic meridian, having its centre on the same level as the indicating needle, and magnetically east or west of it. In this position, which was that chosen for the experiments in which the values of P were observed for the purpose of comparison with the calculated values in the preceding part of these notes, the magnetic force of the compensator has no resolved part in the magnetic meridian at the point where the indicating needle is placed. The sensitiveness of the needle is, therefore, the same in all positions to which the compensator may be turned in its own plane.

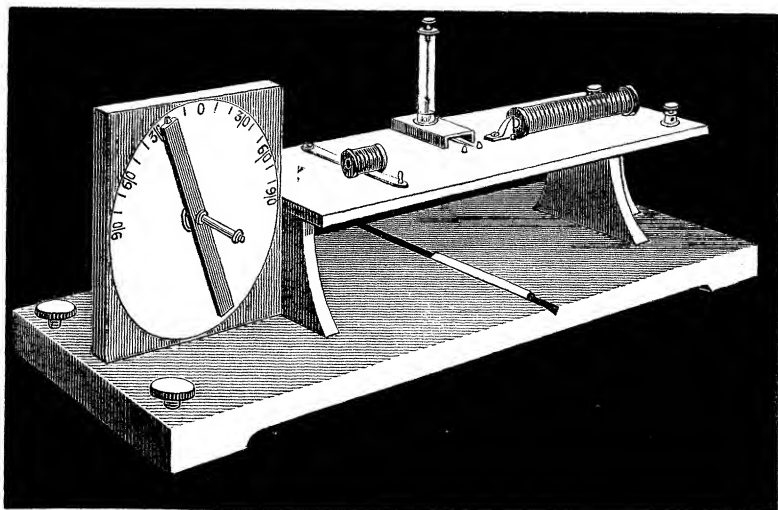
2ndly. A small magnet lying magnetically north and south of the indicating needle is used to astaticise it to the required degree.

3rdly. The compensator is placed so that the distance between its centre and that of the indicating needle is about 4·6 times the half-length between the poles of the compensator. If the latter be a not very thick flat straight magnet, it may be approximately assumed that the poles are about one-tenth of the whole length from the ends, so that no very important error can arise if a distance equal to five times the half-length between the ends of the magnet is selected. When this is done it is safe to assume that for all positions of the compensator within 55° on either side of zero, the magnetic force which it exerts on the pole of indicating needle is proportional within 2 per cent. to the degrees of the scale through which the compensator

has been turned; and is proportional within 1 per cent. for angles less than 45° . For the practical purposes for which the magnetic balance is designed there will therefore be no need of a special calibration table.

4thly. As it is inadmissible to obtain a great range of compensation for both large and small magnetic forces by diminishing or increasing the distance of the compensator from the indicating needle, it is proposed to obtain this range by placing over the compensator a second magnet, capable of rotating round the same axis and having the same length between its poles. A third magnet may be added, similarly, to increase the magnetic moment of the compensator.

Fig. 4.



5thly. In the preceding investigation it has been assumed throughout that the length of the indicating needle was small relatively to r and l , so that the terms of the second and higher orders due to its length might be neglected. This was not the case in the original instrument of Hughes, in which the needle was 5 centims. in length. The use of a very small needle is open to the objection that it is not so sensitive as an index. It is therefore proposed to substitute as an indicating needle one of the type sometimes termed "unipolar," that is to say, having one pole set in the axis of rotation, so that only one pole has a moment of couple about the axis of suspension. A steel needle about 5 centims. long has about 1.5 centims. of its length turned up at right angles, the suspending fibre of silk being attached to the turned-up end; a counterpoise is added behind, and a small additional weight is placed below on a brass wire attached immediately

below the centre of suspension. This needle is placed, with its one effective pole in the axial line of the balance, level with the centre of the compensator.

The balance, with the modifications described, is represented in fig. 4.

February 21, 1884.

THE PRESIDENT in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

- I. "On some Relations of Chemical Corrosion to Voltaic Current." By G. GORE, F.R.S., LL.D. Received February 11, 1884.

The chief object of this research was to ascertain the amounts of voltaic current produced by the chemical corrosion of known weights of various metals in different liquids. The research was also made to throw light upon the conditions which determine the entire conversion of potential molecular energy into external (*i.e.*, available) electric current. The method adopted was to take about six or eight ounces by measure of a corrosive electrolyte and divide it into two equal volumes in two similar glass vessels. Two pieces of metal of equal dimensions were then cut from the same sheet, cleaned perfectly, and weighed. One of the pieces was employed as the positive plate of a voltaic cell in one of the portions of liquid, and the other as a comparison corrosion sheet in the other portion. The negative metal of the voltaic cell was in nearly all cases a large cylinder of sheet platinum, and surrounded the positive one. The positive and comparison plates were wholly immersed in the separate portions of liquid, except that the former had a narrow exterior projecting strip for connexion in a circuit. The current from the cell was passed by means of a small sheet silver anode ($\frac{1}{2}$ inch by $\frac{3}{4}$ inch), and a smaller silver cathode ($\frac{1}{4}$ inch by $\frac{3}{8}$ inch) in a third vessel, through a cyanide of silver plating solution containing the least practicable proportion of free potassic cyanide in order to obtain the maximum amount of silver deposit.

During each experiment observations were made of any liberation

Fig. 2.

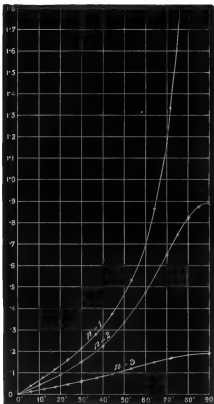


Fig. 3.

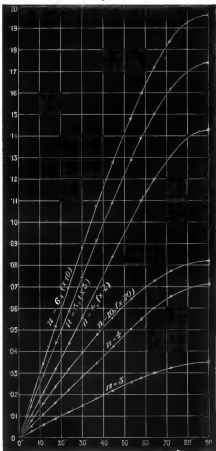


Fig 4

